Journal of Convex Analysis 13 (2006) [final page numbers not yet available]

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The Imaginary Colloquium dedicated to Jean-Pierre Aubin's Friends

This is an abridged English version of the original French version http://lastre.asso.fr/aubin/ColloqueImaginaire-Roscoff.doc focalizing on the mathematical side
Le Colloque Imaginaire du 24 juin 2004 à Roscoff dédié aux amis de Jean-Pierre Aubin

I am very grateful to Pierre Cardaliaguet, Marc Quincampoix and Lionel Thibault and all those, professors, former students, collaborators and friends, for organizing the Roscoff colloquium from June 21 to 26, 2004 and for publishing its proceedings in this special issue of Convex Analysis.

This is the most wonderful gift I ever received, but undeserved, since I should have organized a colloquium for thanking all those who trusted me and contributed to make a mathematician of me: Here are some of the participants I would have invited to such a colloquium, necessarily an imaginary one. Not all, since I lost traces of cryptomnemonic influences which transited too long in my unconscious. I asked the ones I forgot to forgive me as well as those I could have hurt inadvertently or by excessive hastiness.

I am indeed convinced that we are only ephemeral vectors of ideas, including mathematical ones, that we receive, process, somewhat transform before transmitting them to others, through this mysterious alchemy carefully hidden in complicated distilling stills. I guess human brains have innate cognitive capabilities of "doing mathematics" as of speaking, believing and obeying "cultural codes" The environment provides "father mathematics", mother tongues, ideologies and moral codes of the social group. If they are not used enough during the Konrad Lorenz imprinting periods, they fade away. It is only accidentally that I used my mathematical capabilities, genetically that I am talkative (in French only), voluntarily that I am forcing me to doubt as much as I can and lazily that I am more or less abiding by moral codes. In all modesty, I only contributed to this transformation process of ideas. A virtual experiment would suffice: jailed, I would spend my time with history, social sciences, cognitive

sciences, whereas a mathematician, a real one, would make mathematics in the usual sense, as for instance did Jean Leray in the Edelbach oflag during World War II. Chance, and financial necessity, chance of Goddess Tyche, not a stochastic one, led me surreptitiously in the mathematical world. I did succumb to the fascinating seduction of mathematics, to the point of hijacking Ovid's famous verse "*Post coitum, animal triste*'' to transform it into "*Post cogitum, mathematicum triste*", so intense but fleeting the pleasure of understanding, and sharing it by attempting to explain it to other persons. I could not remain insensitive to the beauty of mathematics which, for better attracting us, requires being true. Whenever I am unable to assert the truth of a statement, I declare mezzo voce that I prefer a beautiful false assertion to a true one, but ugly! All the same, everyone to his taste: Beauty is a consensus of a given social group at a given time, and it happened to me that this group was reduced to a singleton! I did not choose this profession, but fully enjoyed it (up to stupid meetings, administration and grading papers), thanks to those I now invite to this virtual colloquium.

1

My first guests are Jacques-Louis Lions and Laurent Schwartz, who left me orphans in 2001 and 2002, whom I have admired and loved as father. Among all the lessons I learned from them, both of human, moral and scientific nature, trying to look for motivations for mathematics and find applications of mathematics in other fields of knowledge was the most important one, and doubtless, the most difficult to learn, the most arduous to implement. For this, and the fact that I owe them everything, professionally and intellectually, this invitation is far from witnessing the wholehearted and devoted recognition they deserve.
Lions, whom I did not see that often during my thesis under his direction, but sufficiently for fueling my morale long enough each time I saw him, confided me that his students (la bande à Lions, or les lionceaux, as other jealous students called us) helped him a lot. I did not understand this statement since we owed everything to this incomparable master. I started to understand it when, in my turn, students made me the honor and the pleasure of trusting me enough for thanking all of them.
Jacques Dixmier triggered for the first time my real interest to mathematics. Gustave Choquet fascinating me during his course in topology where his wide movements with his hands replaced the chalk on the blackboard. Jean-Pierre Kahane made me the honor to seat my thesis committee and our ways crossed several times. I loved and appreciated the course of Madame Dubreuil-Jacotin, this pioneer in the male mathematical world of this time. But it was Tyche who placed Lions on my way, when, at the time where mathematics seldom started to interest an undergraduate student still torn between history and mathematics and mainly involved in underground movements helping deserters of the Algerian war, I attended only once out of curiosity a seminar at École Normale Supérieure in 1960 where I was under the charm of young energetic man full of communicative enthusiasm with a southern accent who, I still remember,

spoke about interpolation spaces. I soul not understand anything, but this was still engraved on my memory. One year later, arriving at Électricité de France (EDF), I heard that a Professor Lions just gave a course on Numerical Analysis. I made the link, and wrote him at Nancy. The day after, I received an answer, positive: "read my book *Équations différentielles opérationnelles*, we shall meet in six months". This book was just out of press, the first one of a long series of monographs too early interrupted.
I read it, I saw him, I was won over.
This was in 1962.
Actually, everything really started with a confidential group of students who met weekly in 1960 among whom Salah Baouendi, Gisèle Fritz, Charles Goualouic (who left us much too early) and few others. Lions arrived in Paris in 1962 bringing with him Jean Céa and his optimistic sense of entrepreneurship and Pierre Grisvard and his generous seriousness, who also left us. I next met Pierre-Arnaud Raviart at EDF, were we made between 1962 and 1966 a quite turbulent team in a public enterprise which knew for a long time how to develop first class applied research worth of its name. Lions's seminar on numerical analysis attracted a new generation of students, among whom Pippo Geymonat, Roger Temam and Haïm Brézis.

2

Next, in 1986, my discovery of an America who had nothing to compare with the one we now know, first at Madison, then at Purdue. The wind of liberty of minds and behaviors blew in all aspects of life. One could measure it with the yardstick of the hems of skirts of students at Purdue : At ankles, when I arrived in 1967, at the global minimum two years later. At Purdue, François Trèves did exert on me a tremendous beneficial influence. I also met Jerry Siegel who will find later a simple and elegant proof of Ekeland variational principle. At Madison, I benefited from the advice of John Nohel, Ken Smith and Laurence Young (first docteur Honoris-Causa of Dauphine), "melting" all qualities that French people grant to their English neighbors, at Chicago, of Felix Browder, and of so many other American friends who made m love this America. I spent the ten summers of the 1970's at the Mathematics Research Center of Madison. It was the best model of a Research Center I ever had a chance to visit, with recurrent randomly overlapping visits of mathematicians of all horizons, mathematical as well as geographical, melting pot of scientific cultures. A disappeared model. I mathematically blossomed there. Among the many interlocutors I met, I have a special debt to Amnon Pazy, Mike Crandall (who mentioned me Nagumo's theorem when I started investigating viability), Steve Robinson, Edouardo Zarantonello, who introduced the theory of monotone operators (without forgetting George Minty and his famous "trick"), which became so successful, particularly in France when Lions introduced it and Haïm Brézis developed it. The summer of 1969 put a brutal end to my first mathematical period devoted to functional numerical analysis of some classes of partial differential equations.

3

For some weeks before arrived a letter of Lions to whom I confided my real vocation, social and human sciences: Dauphine was just created for developing economic and management sciences. Farewell to USC, which I was about to join at the invitation of Henry Antosiewicz. During this summer of 1969, I was looking for an economist who would get me initiated quickly to the mysteries of his science. I had the privilege to meet a thinker too independent and original to be recognized at his true value, Dick Day. He was for example the very first one to introduce the notions of chaos in economics. I don't know whether he was ever satisfied with his student, but I am mostly grateful to this master and friend. Poet, curious, cultivated, we were made to get along. I also met this very summer Alain Bensoussan, whom I did not know. I told him about Dauphine, he answered me that he shared the same tastes than mine. The difference was that he was competent (he also graduated from ENSAE) and was particularly interested to management. We decided to team up, and this team still lasts
I did not recognize France and its universities when I returned in September 1969. Pierre-Marie Larnac and Francine Roure welcomed me there and actually, played a fundamental role in the creation of mathematics in Dauphine (which was not planned in the university project). Pierre Tabatoni, the designer of this new university, was in sick leave at the time, but still was advising this initial team. He joined Lions and Schwartz in my personal Pantheon. The wonderful anarchist atmosphere which did not last long after 1968, the absence of traditions allowed us to invent freely what we had in mind. We did not take care of the mass and maze of bureaucratic regulations that we did not worry to learn anyway, so that we could disobey, but deprived of the pleasure to know it! Alain Bensoussan then joined me, and next Patrick Saint-Pierre, a former student of Roger Temam. We started a long lasting collaboration, still very active, Patrick bringing an applied and numerical touch to my too abstract orientations. Ivar Ekeland attracted me right away after a talk he gave to the Lions seminar when I met him for the first time: At the colloquium dinner, I invited him to join us at Dauphine. The same scenario happened few weeks later with Luc Tartar, who left Dauphine for Orsay because he really preferred physics to economics. He was replaced by Pierre Bernhard, with whom I continue to collaborate.
Recruiting was stalled during the seven years of Giscard d'Estaing's presidency : professor positions had to be negotiated one by one, teaching assistant positions were brutally cancelled without any replacement system.. Pierre Tabatoni was instrumental to recruit Hervé Moulin and Jean-Michel Lasry. Pierre-Louis Lions replaced in 1981 Pierre Berhard who joined INRIA to create the site of Sophia-Antipolis. I was by the time unsuccessfully trying to create privately sponsored chairs for inviting foreign colleagues for short, recurrent and overlapping visits. Knowing that, the late Recteur Jean Prieur transferred to Dauphine one of the seven chairs of the Ville de Paris, on which we could recruit Yves Meyer, then professor at École Polytechnique. Except Yves Meyer who is my age, they did not finish their thèses de doctorat d'État. My wish to invite them to Dauphine has been instinctive and sudden (see Aubin J.-P. (1995) La genèse des mathématiques de la décision à l'Université Paris-Dauphine, Gazette des

Mathématiciens, 65, 39-45 for more details). I know that I will sound a sinner by the mores of the time, but I never read a CV, except once where the CV was wonderful but the choice wrong. I am doubtful of all these formal evaluation procedures which are by now so popular, which evaluate scientific production rather than the producers. Numerical indexes (quotation index, number of publications) will never replace (wo)man to (wo)man global evaluation. After all, history confirmed that these choices were not so bad.

Sanjoy Mitter was the first lecturer of the Aubin-Bensoussan-Larnac seminar opened in 1970, which became the colloque du CEREMADE until it was closed in 1996. One, soon after two weakly seminars followed. One "vertical" department, from first year to doctoral studies, started in October 1970 with 12 first year students recruited through an ad in the newspaper "*Le Monde*" two months before we got the authorization from the ministry of education, a research center, CEREMADE, two graduate programs, attended by student who became famous, were launched. Everything went so fast! I abandoned all my various administrative responsibilities in Dauphine in June 1981 and spent the next three years trying to protect Institut Henri Poincaré (IHP) from vultures and to keep in the hands of mathematicians with the help of Jean-Pierre Bourguignon and Jean-Marie Schwarz. I was happy to say that CEREMADE existed only as a letter head, so that he could not be destroyed. I thought it was a joke, but I was right: once it started to exist bureaucratically, the CEREMADE of my dreams fade away from 1986 to 1996. I keep only in my memory those fifteen first years of CEREMADE' life. In retrospect, I would never do it if I had to do it all over again.

Actually, I realize now that the new information and communication tools (Internet, chip telephone and travel) allow us to invent networking structures, not necessarily anchored in a geographical and/or instructional location, flexible structures allowing specialties of scientists to evolve in a differentiated way, much faster than the time of carbon paper with which I began.

4

During this period, Gérard Debreu and Edmond Malinvaud were kind enough to guide our first steps in economics and to introduce us to their younger colleagues of the time, among whom Claude Henry, Thierry de Montbrial, Paul Champsaur, Roger Guesnerie, Gérard Fuchs, Alan Kirman, Werner Hildenbrand, constituting this kernel of mathematical economists orbiting around ENSAE. This school became a close associate of Dauphine, collaborating to a common doctoral program and recruiting as professors Hervé Moulin, Bernard Cornet and Georges Haddad.

5

This Dauphine adventure went hand in hand with the one of École Polytechnique: Laurent Schwartz invited in October 1969 Charles Goulaouic and me. From 1970 to 1974, Schwartz asked Thierry de Montbrial (next replaced by Roger Guesnerie) and me the teaching of the four-month optional course on

mathematical economics. The first one to attend this course was Schwartz himself. This was the most difficult exam of my life. I met Schwartz before I even knew he was a mathematician. I was involved in the clandestine networks who helped people who deserted for ideological reasons, since neither the Catholic Church nor the French Communist Party agreed to support them. Two of my courageous friends were arrested, and I was advised to ask Laurent Schwartz for help. He did not hesitate at all. He was a "lay saint", as Jean Daniel titled a paper in "*Le Nouvel Observateur*" after he succeeded with the complicity of Michel Broué and other mathematicians, Plioutch was freed and came to France. Those were the ones who inspired the help we tried to bring to Alexander Ioffe since of 1975, with the collaboration of Richard Vinter at the other side of the channel.

6

This kernel of colleagues attracted young doctoral students, Hervé Moulin, who was actually a "researchmate", since in 1969, I knew nothing on mathematical economics, a field to which he was much more faithful than me. He was the very first docteur d'État of Dauphine, the second one being Jean-Michel Lasry who was working with Ivar Ekeland. Next came Bernard Cornet, who studied continuity properties of set-valued maps, fixed point theorems motivated by the "joueurs de boules de la Porte Dauphine", and next extended Claude Henry's papers on dynamic planning processes which attracted the attention of Steve Smale. Steve Smale has above all influenced Yves Balasko who participated regularly to the activities of CEREMADE. Georges Haddad proved the main viability theorems in the framework of differential inclusions with histories and constraints on histories of evolutions, existence of periodic solutions with Lasry and second-order viability problems with Cornet. We did not know that Serge Gautier proved this theorem in the case of differential inclusions. After being elected the youngest university president (of université Panthéon-Sorbonne) and at the head of the board of university presidents, next special advisor of the director of UNESCO, and, since April 2004, head of the highest education at UNESCO, he continued to work on differential inclusions with history, introducing "Clio derivatives" for defining Hamilton-Jacobi equations with history and contributing to the study of hybrid systems. Daniel Gabay participated to the activities of CEREMADE after his return form Stanford, collaborated with anyone and above all, with Hervé Moulin. Gérard Lebourg worked with Ivar Ekeland, taught me a version of Ekeland's variational principle that I did not know, and participated to the genesis of the inverse function theorem for set-valued maps which was later so well extended and developed by Halina Frankowska, topic kept alive by Alexander Ioffe, Terry Rockafellar, and others, as the paper by Asen Dontchev and Marx Quincampoix shows. Jean-Charles Rochet, a student of Hervé Moulin, completed this group of young mathematicians at the end of the 1970's.

7

Around this minuscule but lively group orbited many regular and active guests, among whom Umberto Mosco, Arrigo Cellina, and their Italian friends of the "Tchérémadé", Maurizio Falcone, Italo Capuzzo-Dolcetta, and later, Nicoletta Tchou and Paola Loreti, among many others. This is at this time that I met Terry Rockafellar (at Chicago, in 1969 and above all, at Varena, with Werner Hidenbrand), who advised us to invite Franck Clarke. They played an important role in increasing the scientific potential of CEREMADE, together with Dick Day, Henry Antosiewicz, Guillermo Owen, Shmuel Zamir and his Israeli colleagues, etc. Hector Sussman replaced me in Dauphine during one year to pursue his crusade against the abuses of "catastrophism ". I did not have time to be brainwashed with differential geometry by Claude Lobry and him, bypassing the luxuries of tangent spaces of aristocratic smooth manifolds to thrive directly in the uncomfortable but affordable slums that are the tangent cones to proletarian subsets introduced by Francesco Severi, Georges Bouligand and Gustave Choquet. Shi Shuzhong has been one of the first Chinese mathematicians to arrive in Collège de France, where Lions invited him and advised to visit CEREMADE. He since became a collaborator and a friend, with whom we shall adapt LASTRE's activities in China to comply with the global world economy! He was the first to extend viability theorems to parabolic partial differential inclusions. Back in China, he built research centers inspired by CEREMADE in Tianjin and next, to Beijing. He organized in 2000 a course on financial mathematics in the framework of CIMPA (headed by Claude Lobry, who succeeded to Pierre Grisvard and Jean-Michel Lemaire). Ky Fan (second docteur Honoris-Causa of Dauphine), the inequality of whom I was and still remain an untiring proselyte, made me the honor and the pleasure to adopt me. It was at the occasion of a colloquium organized in 1984 at Santa Barbara by Stephen Simons (who I knew before thanks to an elegant and economical proof of a fixed point theorem), that I talked for the first time of viability kernels. I did not realize at the time that this concept will play a role in viability theory analogous to his inequality in nonlinear analysis.
These visitors taught me economics, optimization, nonlinear analysis, differential equations and inclusions of which I ignored everything.

8

I owe Jean-Jacques Moreau the pleasure to read the notes of his course in Collège de France recently republished for his eightieth birthday. A long lasting friendship and implicit collaboration started with Terry Rockafellar and Roger Wets, at IIASA and Dauphine, which is still lasting. It allowed us and many others to explore together the many roads of set-valued analysis just before and soon after 1980. Our discussions have been frequent, fruitful, I witnessed the long genesis of their monograph "*Variational Analysis*" (Springer). They are the ones who made me conscious of the importance of the epigraphical and next, graphical approach allowing functions and set-valued maps to inherit the properties of sets and of their tangent cones. I benefited all along these years of their works which did not cease to enrich my thinking, even though this did not

lead to common publications (often the tip of the iceberg of the underlying long term maturation of fundamental ideas and techniques). I owe them this part of my mathematical activity. I do not forget Pierre-Jean Laurent and Jean-Luc Joly, the colloquium of Saint-Pierre de Chartreuse, all the "south-west optimizers", Charles Castaing, Michel Valadier, Pallu de la Barrière's school, Jean-Paul Penot, Jean-Baptiste Hiriart-Urruty, Lionel Thibault, etc., too many to be all quoted here. I met Roger Wets at the colloquium of Murat Le Quaire, where, after having presented my first clumsy attempts to what will become viability theory I met Heddy Attouch and Alain Damlamian, proselytes of maximal monotone operators. Being deterministic in the sense that they produce unique evolutions, they were not fit to encapsulate Darwinian evolution contrary to nonmonotone differential inclusions. During this period, I benefited of a wonderful one-year escapade at the CRM of Montréal in an awaking Québec with the election of the Parti Québécois. Michel Delfour and Andrzej Manitius taught me their work on partial differential equations with delays. Heddy Attouch and Frank Clarke did organize the congrès franco-québécois à Perpignan (dedicated to Jean-Jacques Moreau) which symbolized my farewell to static optimization and nonlinear analysis.

9

On the mathematical front, I started to find my way to my third (and last) mathematical period: "viability" began to come into my mind. I woke up a morning of the spring of 1974 with the idea of translating the title of the famous Jacques Monod book, "*Chance and Necessity*" by $x'(t) \in F(x(t))$ and $x(t) \in K$, without knowing nothing neither about the first term until I met Arrigo Cellina who told me that differential inclusions were introduced by Marchaud and Zaremba in the early thirties nor on the second one before Mike Crandall mentioned me that the issue was taken up by Nagumo in 1942 in the framework of differential equations.

This happened after a slow maturation triggered by my personal interests in economics, biology and cognitive sciences. Won over by his book "*Tout empire périra*" which played a crucial in the genesis of the inertia principle and heavy evolutions, Jean-Baptiste Duroselle welcomed me with a kindness equal to his historical erudition on which he based his quest for historical regularities. I met him too late, he left us too early. The message was the same than the concept of punctuated equilibrium of Eledredge and Gould appearing at the same moment. Julian Jaynes "*The origin of consciousness in the breakdown of the bicameral mind*" was also very influential in shaping my early ideas. Reading "*The self and its brain*" by Eccles and Popper blew up my mind, triggering an exchange of letters with Karl Popper, where I suggested another classification of his three worlds, adding a fourth one. This is in a 1982 working paper of IIASA that I suggested that far from being coded in mythical synaptic weights, information was coded by maintaining cyclic circuits of neurotransmitters from one neuron to another, subjected to recognition mechanisms, complementing the same kind of information coded by hormones but by faster and specific means. I

mention it because, despite a quite general skepticism, I still persevere. I started to discuss this issue with Ivar Ekeland and Frank Clarke, who were not that much interested, with Arrigo Cellina who taught me differential inclusions, with Georges Haddad who proved the basic theorems and with Patrick Saint-Pierre who started to throw his numerical look.
This opening of a new road did not seem to interest many persons at that time. So I decided to withdraw in my own shell until the time would be ripe, if it would ever happen. I made advances in my own perception of the world, and that was enough for my scientific happiness. This was quite inoffensive any way, except for my students and collaborators guilty of following such a track, particularly to Halina Frankowska and Patrick Saint-Pierre victims of an intolerable and incomprehensible injustice.
I felt myself at last ready, twenty five years after a slow start.

10

Halina Frankowska, a former student of Czeslaw Olech, was invited at Dauphine by Jean-Michel Lasry in 1982. She proved at that time first-order and higher-order inverse function theorems that she used for deriving variational inclusions of partial differential inclusions allowing her to study issues in local controllability. Pierre-Louis Lions asked her to make the link between general gradients used in nonsmooth analysis and in the theory of viscosity solutions of Hamilton-Jacobi equations : This was the starting point of one of the research programs that she investigated, providing an epigraphical and geometrical approach to this topic, which continues to find fruitful applications to the study of first-order partial differential equations inclusions, including Burgers' type equations and is at the origin of characterizations of viability kernels and capture basins. She devoted to the theory of Hamilton-Jacobi equations a long series of papers, some of them with Piermarco Cannarsa and/or other collaborators.
The mathematical side of our relation is known, the private side will remain unknown.

11

I spent more and more time at IIASA during the 1980's, at the "Systems and Decision Sciences" Project, under which applied mathematics could develop at the time, oriented towards control theory when Alexander Kurzhanski headed this project. This is where the concept of viability kernel popped up thanks to some numerical experiments by Zenon Fortuna, investigations of second-order viability problems and current investigations with Czeslaw Olech and Halina Frankowska on controllability of convex processes. IIASA offered our research group during fifteen summers an ideal working place replacing the defunct Madison MRC, allowing me to interacted with many visitors and above all, with young scientists of the wonderful YSSP (Young Scientists Summer Program). I met Karl Sigmund and Josef Hoffbauer and was attracted by their work on replicator systems and games, Don Saari, who impressed me by his parallel study of celestial mechanics and social choice, as well as his treatment of paradoxes

and his work on "chaos à la Saari". The colloquium "*Dynamics of macro-systems*"we organized at IIASA with Karl Sigmund and Don Saari was an important landmark in the development of my research. We met Pravin Varaiya, Art Krener and Chris Byrnes, Alberto Isidori and many other specialists of control. The "zero-dynamics" and out viability kernels had the same genes. I also met George Leitman at IIASA, and it was admiration at first sight, and thus, a doctor Honoris Causa of Dauphine. Later, in 1999 and 2000, Halina and I were visiting Roger Wets and Art Krener at Davis and Shankar Sastry invited us at Berkeley. His students, among whom Claire Tomlin and John Lygeros, surrounded us at lunchtime and asked us a thousand questions, speaking about hybrid systems I never heard of. Patrick Saint-Pierre and I worked on these papers during the summer of 1999 to discover, as Shankar suspected it, that viability techniques could play a role. Nicolas Seube, Marc Quincampoix and his student, Éva Crück, Patrick and me collaborate with John Lygeros, Shankar Sastry, Claire Tomlin and in 2001, a student of her, Alexandre Bayen, a former graduate of École Polytechnique and now professor at Berkeley. Alexandre learned quickly and deeply the mathematical and numerical aspects of viability theory to apply them to Burgers' equations and variants useful in transport theory, by observing that the graph of the solution to such a system of first-order partial differential equation with initial conditions, boundary conditions and viability constraints is a capture basin.
In 1998, neither hybrid systems nor first- order partial differential equations were in my research agenda, which do not exist anyway. I am taking this opportunity to underlie the lethal consequences of evaluating research projects rather than former research activities, since by definition, a real scientist cannot know what he/she will discover next. If Gosplans did not work to plan industry, research projects cannot achieve the role that "science bureaucrats" who are swarming along the increasing number of funding agencies. Instead of forcing scientists to waste their time to write research projects which can please the scientific bureaucrats who fund their research, and thus, necessarily abiding by mainstreams, funding agencies should regularly and rarely, every four to five years, analyse carefully past research activities, not only productions in a given current, but the originality and the ability to seize opportunities, among many other criteria. It would then suffice to use this evaluation to grant him/her the needed financial support for the next 4/5 years, without asking the impossible forecast of what he/she will do. In the worst case, five years of support will be lost. Nothing compared to the waste of time, energy and money that the unfortunate "macdonaldization" of sciences imposes worldwide with a costly inflation of pseudo-scientific automated evaluation procedures of not always relevant qualities.

12

At the end of the 1980's, the socialist government created doctoral grants which allowed us to recruit again doctoral students. The seminar "Viabilité-Jeux-Contrôle" organized with Halina and Patrick Saint-Pierre in 1988 attracted

new students. After giving birth to the informal Réseau de Recherche Viabilité-Jeux-Contrôle, this seminar is still active at Institut Henri-Poincaré. Olivier Dordan explored the domain of qualitative physics and analysis, which he exposed in his 1995 book "*Analyse Qualitative*" (Masson). Marc Quincampoix followed, working on viability approaches to differential games with the help of Pierre Bernhard. He discovered the fundamental fact that boundaries of viability kernels are "barriers" and built a first bridge with classical methods going back to Isaacs. Nicolas Seube made his thesis at Thomson-Sintra on neural networks for UWV (Under Water Vehicles), and continued to develop this project from mathematics and simulation to prototypes of gliders built at ENSIETA, which sail in Brest's roadstead. He won the 1991 ICIAM prize of best young researcher paper. Pierre Cardaliaguet studied the Wazewcki property and invented the discriminating kernels and the algorithm to find them in differential games, continuing by studying front propagation problems and many other problems. Marc and Pierre teamed up with Patrick Saint-Pierre, who, with the contribution of Philippe Lacoude, designed the Viability Kernel Algorithm, for writing a series of "*CQSP*" papers on algorithms and differential games. Nicolas, Marc and Pierre transformed Brest into the new Mecca of viability, as this colloquium held in Roscoff shows. Nathalie Caroff worked with Halina on Riccatti equations for nonlinear control problems and is now at Perpignan, where Ouana Serea, a student of Marc Quincampoix, joined her.
Luc Doyen joined Dassault-Aviation to work on visual control and started to prove the first theorems on mutational analysis. This field emerged from the confrontation of splendid papers by Jean Céa, Jean-Paul Zolésio and Michel Delfour on shape derivatives in shape optimization, of "funnel equations" studied in former Soviet Union by the Panasiuk brothers, Alexander Kurzhanski, Vladimir Veliov and Istvan Valyi (who further developed them at IIASA) and my tychastic meeting with Michel Schmitt who gave me his thesis on mathematical morphology in which I discovered that late the pioneer work of Georges Matheron, Jean Serra and their school. I immediately taught mathematical morphology in my graduate course, and this triggered Juliette Mattioli and Laurent Najman to start their research on this topic under the joint supervision with Michel Schmitt at Laboratoire Central de Recherche de Thomson CSF, now Thales. Juliette's thesis ended up with the 1993 book "*Morphologie Mathématique*" (Masson) she wrote with Michel Schmitt. Next Anne Gorre completed these results on mathematical morphology by theorems on morphological equations governing the evolution pairs of sets preserving a nonempty intersection or contained one into the other along their evolution. Luc, Juliette, Laurent and Anne laid down foundations that are taken up by Alberto Murillo at Cartagena on viability multipliers for morphological equations, Thomas Lorenz, at Heidelberg, who wrote a remarkable thesis on much more general morphological equation for morphogenesis, and Sylvain Rigal, a student of Marc Quincampoix. Daniel Gabay told me about results by Jean-Pierre Quadrat, Marianne Akian and Stéphane Gaubert on Maslov measures and max-plus algebras, taken up then by Juliette Mattioli and Pierre Bernhard. They allowed Olivier Dordan to make a link with fuzzy differential equations and, more recently, Francine

Catté to use lattice properties for characterizing viability kernels and capture basins.

13

Meanwhile, Halina and me convinced the manager of a European program to build between 1989 and 1992 a graduate teaching program which was the best pedagogical experience of my career. It offered a portfolio of 20-hour doctoral courses taught during one week in secluded areas by a team of 2 to 3 lecturers. Such an organization allowed us to diffuse innovative courses to students coming from all Europe, instead of addressing a group of students of one university only. During these courses, students from all countries could interact permanently among themselves and with the lecturers in residence with them, and thus, available for private questions. The productivity of such courses is worth of the higher costs, and I dream that this experience could be broadened and extended at the European level. By not granting degrees, such a program does not compete with doctoral programs of universities: They are free to negotiate case by case how to reward locally the students attending such courses.
In a similar manner, for bringing new techniques which would take years to reach the potential users in industry of financial institutions by teaching them weekly in each university, mathematicians should try to contact these potential users and to convince them and their universities to design an analogous format for teaching engineers or professionals courses designed and taught relevant to and adapted to applications.
Noël Bonneuil and Katherina Müllers (from Sarrebrück) attended one of these courses, and since them, Noël is using viability tools for his investigations in population dynamics, sociology, anthropology, and Katherina Müllers worked with him on population dynamics, with Jean Cartelier on a Keynesian model and studied "inertia cascades". 14.

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Jerzy Zabczyk mentioned me in 1996 that the problem of pricing an option was closely related to viability kernels and capture basins. He actually introduced and studied these concepts in the framework of discrete systems under different names. This was enough to attract my attention to a problem which did not interest me, contrary to real economics, whatever the reasons that only a psychoanalyst could discover. Actually, we did not know at the time under which conditions the capture basin of a target viable in an environment under a differential inclusion or a dynamical game was closed. At the same time, with Noël Bonneuil, we noticed that the graph of the solution to McKendryk equations (age-structured) was a capture basin, and, at the same moment, in the summer of 1999, Patrick Saint-Pierre and I realized that the viability kernel of an impulse system involved the concept of capture basin, too. Hence the pressure became so great that we discovered that a simple sufficient condition was that the viability kernel of the complement of the target in the environment is empty.

So, the missing tool, the capture basin, could join its cousin, the viability kernel, to solve many problems the solutions of which can be expressed in terms of viability kernels and capture basins. Both could be embedded under the umbrella of viability kernels with targets introduced earlier by Marc Quincampoix and Vladimir Veliov. This is at this point that Francine Catté proved that the viability kernel with target is the unique bilateral fixed point between the target and the environment of the viability kernel regarded as a function of two variables, the environment and the target. She used an approach inaugurated by Juliette Mattioli et Luc Doyen, where Matheron's results took an unexpected place providing an algebraic framework allowing us to unify Saint-Pierre's viability kernel and capture basin algorithms and Cardaliaguet's algorithm for computing the discriminating kernel.
Dominique Pujal could then use capture basins to study the valuation and the management of portfolios replicating a whole variety of options both for the Cox, Ross and Rubinstein framework and dynamical game against nature. She devised with Patrick Saint-Pierre the Capture Basin Algorithm computing it, and thus, allowing to price the value of a portfolio subjected to all kinds of constraints, bypassing (but recovering) the Black and Scholes equations. We suggested to coin the Greek name for chance, tyche, instead of using the word perturbations, and to call tychastic systems the ones too often known under the oxymoron "deterministic uncertain systems". This was further justified when Halim Doss told us that the Strook-Varadhan theorem implied that stochastic viability was a very particular case of tychastic viability.
We them saw viability kernels and capture basins everywhere. When environments and targets are epigraphs of extended functions, they are epigraphs of solutions to Hamilton-Jacobi-Bellman-Isaacs in the Frankowska sense, that she proved by duality to be equivalent to viscosity solutions if they are continuous, to solutions in Barron-Jensen/Frankowska sense if they are only lower semicontinous. When the environments and targets are graphs of set-valued maps, they are graphs of solutions to systems of first-order partial differential equations or inclusions. This is the case of solutions to McKendryk, so that allowed Noël Bonneuil, Frank Maurin and me could bypass the semi-group approach, and of solutions of Burgers, equations, which allowed Alexandre Bayen and Patrick Saint-Pierre to give a meaning to set-valued solutions (with shocks) to these equations, regarding single-valued solutions (Rankine-Hugoniot, entropy) as almost everywhere selections of the set-valued solution, whereas earlier Halina Frankowska proved the existence of a global solution with non empty values to the centre-manifold problem for nonlinear systems, the graph of which being a viability kernel. Everything could take place in a unified framework thanks to the coordinated efforts of mathematicians of all generations and many fields outside mathematics constituting the informal "réseau de recherche Viabilité-Jeux-Contrôle".

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We started with Giuseppe Da Prato and Halina a long a deep collaboration on stochastic viability issues, although I am not convinced that the standard

Kolomogorov/Wiener mathematical translation of uncertainty answers the challenges of encapsulating uncertainty for systems involving living beings. Giuseppe Da Prato is associated in this adventure with Jersey Zabczyk and Halim Doss. Halim having proved that stochastic viability is a particular case of tychastic viability, he showed that for subsets defined as the level set of a smooth function, the second-order viability conditions were equivalent to first-order ones. This has been extended to any closed subset by defining a concept of contingent curvature (using contingent derivatives of the normal cone map) and extended to control systems Halina et Giuseppe Da Prato, whereas Rainer Buckdan, Pierre Cardaliaguet, Marc Quincampoix and Catherine Rainer offered other characterizations. When environments and targets are epigraphs of extended functions or graphs of maps, viability kernels or capture basins are epigraphs or graphs are solutions to second-order partial-differential equations: this is being studied by the Brest group.

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Khachayar Pakdaman started in 1992 an interdisciplinary weakly seminar on networks of oscillators during 3 years, where biologists, physicists and mathematicians exposed their different point of views and techniques. This interest resurrected recently with Jean-Pierre Françoise, Claude Piquet, Catherine Doss and their team studying bursting evolutions of nervous influx, with whom we started to collaborate. What was missing at the time was the 2004 book "*Oscillations en biologie*" (Springer) by Jean-Pierre Françoise, which presents the relevant techniques of dynamical systems. Actually, much earlier Antoine Danchin and Jacques Demongeot influenced my approach to cognitive sciences. The program "Dynamics of Complex Systems in Bio-Sciences" of the European Science Foundation funded the organization of a series of meetings organized by Willi Jäger, Odo Diekmann, Jim Murray, Mimmo Iannelli, Karl Sigmund, Jacques Demongeot and me. Among them, the Fontevraud meeting in June 1994 was the first one really devoted to viability and its applications. I met Yves Burnod at another meeting of this program, with whom I discovered that we shared the conviction that many of biological enigma can be deciphered when they appear for the first time during phylogenesis, when new genetic combinations trigger in a non teleological way the apparition of a new organ which may find a use that selection keeps or not. Yves showed me at one of our weakly lunches the atlas of fetus brains showing that before the 27th week of pregnancy, the brain of human fetus was the same than the one of chimpanzee, with the sketch of a sulcus in the visual area, which was closing and delimitating the Wernicke, available to specialize in the elaboration of concepts in the human brain. To a small cause, huge consequences. Halina joined Yves and his colleagues since she started to be involved in the study of learning processes, bringing with her a newcomer, Sophie Martin, Together with the philosopher Joëlle Proust and the psychiatric Bernard Pachoud she organizes at CREA an informal working group on cognitive sciences attended by Yves Burnod and me.

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Pierre Bernhard was my guru for differential games and related matters. Our long term collaboration involved Odile Pourtallier, Alain Rapaport, Mabel Tidball on one hand, Marc Quincampoix, Pierre Cardaliaguet, Halina Frankowska, Patrick Saint-Pierre on the other hand, an is still active. I could draw on my past activities on static games, involving the cooperation with Jacqueline Morgan and her Neapolitan accomplices, and their colleagues. I get acquainted with the specialists of differential games, particularly with Vladimir Gaitsgory, Alain Haurie at Geneva, Michèle Breton and Georges Zaccour at Montréal.

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Younger persons continue to join the viability group, thanks to grants of the European Union, the last one being coordinated by Marc Quincampoix: Aurelien Cernea (Romania) worked with Halina on optimal control under sate constraints, Alberto Murillo-Hernandez (Cartagena) who also worked on second-order viability problems, Thomas Lorenz (Heidelberg) on mutational equations, Klaus Eisenack (Potsdam) on qualitative physics, Giuseppe de Marco and Mariella Romaniello on dynamical games arising in political sciences, Telma Bernardo on "inertia functions" and their applications to climate models.
Sophie Martin, who graduated from École Polytechnique, joined our group three years ago, and worked with all of us: Halina, on learning processes, Patrick Saint-Pierre, for improving many aspects of the viability kernel algorithm, and me just for talking. She defended her thesis in June 2005 on resilience in ecology and lake eutrophization, presided by Yves Meyer, who was lavish with compliments.
This story is then a little like the French movie "*La Ronde*", except that the circle is a helix: Only the projection is periodic, so that something remains invariant mathematics. Sophie is young, I am old, but the story continues.

Jean-Pierre Aubin

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Laboratoire d'Applications des Systèmes Tychastiques Régulés
July 1st, 2005