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Cognitive Systems

We start from the fact that an organism must **adapt to environment constraints**, by perceiving it and recognizing it through **“metaphors”** with **“conceptual controls”** .

Even if we assume that the basic principle is simple (biological communication), its structure became more and more complex in the course of evolution. Indeed, the improvement consists in adding new structures to old ones, scarcely destroying them.

The understanding and mastering of the basic structures may someday allow a possible computer translation by economizing the useless part of complexity which results from (non teleological) biological evolution, retaining only what was relevant to allow adaptation.

This problem of adaptation is not taken explicitly in most studies of neural networks.

A cognitive system is described by its state and a “**conceptual control**” . The state, called the “**sensory-motor state**” , is described by :

- **the perception of the state and the variations of the environment** on which the cognitive system acts.
- **the state of cerebral motor activity** of the cognitive system, which guides an individual’s action on the environment.

Speculative assumption:

The regulatory control of the cognitive system, called “ conceptual control,” is **an endogenous cerebral activity** which is not genetically programmed, but **acquired by learning and recorded (stored) in the memory**. Its purpose is to “**interpret (process)**” **the sensory perception of the environment for action**.

- **A Recognition Mechanism**, under genetically programmed evolution, which selects metaphors between the state and the variation of the environment on one hand, and the conceptual controls, on the other hand, by matching at each moment the sensory perception conceptual control.
- **Viability Constraints**, which restrict the cognitive system to “adapt” to the environment by transforming it and consuming scarce resources.
- **An Action Law**, which governs the evolution that results from the action of the cognitive system.
- **A Perception Law**, which governs the evolution of the cerebral motor activity from the sensory perception.

Instead of imposing a priori learning rules, we shall select them according to a given principle (such as the **inertia principle**) among **learning rules consistent with the viability constraints** provided by the learning process.

The Learning Process which is a feedback set-valued map (non-deterministic feedback) associating a **set of “learnable” conceptual controls with each sensory-motor state**, which is consistent with the viability constraints and the recognition mechanism.

One can prove that under adequate mathematical assumptions, **given the recognition mechanism, the viability constraints, the action and perception laws, there exists a largest learning process** among which one can choose specific learning rules, and study further its properties.

We postulate also that the evolution of conceptual controls obeys a **“inertia principle”**, stating that whenever a conceptual control **“works”** (i.e., keeps the evolution of the cognitive system viable), **then it is maintained.**

This principle is implemented by selecting **“heavy viable solutions”**, which minimize at each instant the velocity of the conceptual controls. This leads us to associate with any conceptual control a **“sensory-motor cell”**: When the state of the system reaches such a sensory-motor cell, it can evolve inside it while being regulated by its conceptual control which remains constant (It is a property which explains the concept of **“punctuated equilibrium”** proposed by Elredge and Gould in paleontology.)

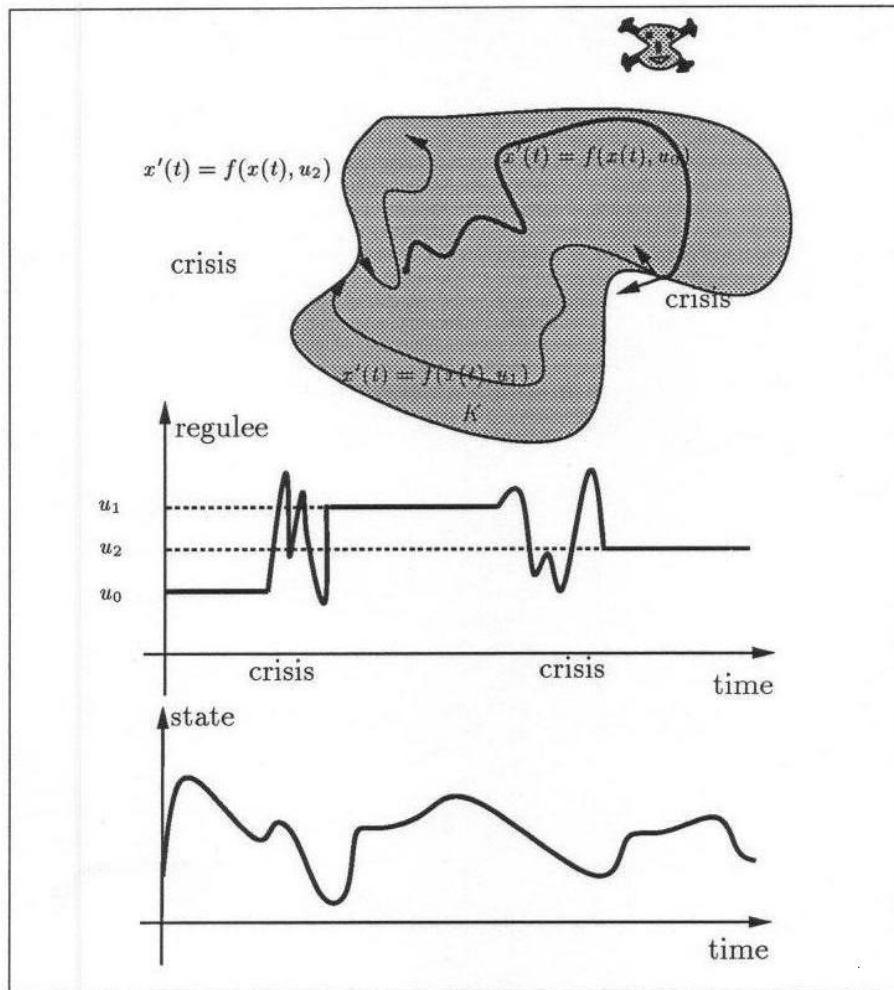
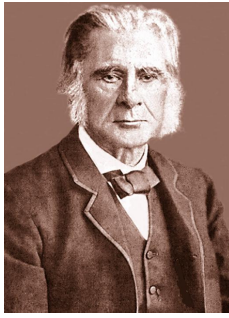


Figure 1: *Punctuated Evolution*. Starting from x_0 with the constant regulon u_0 , the solution evolves in K until time t_1 , (first *punctuated equilibrium phase*) when the state $x(t_1)$ is about to leave K and when the constant regulon u_0 must evolve. Then a *crisis* happens during which velocities also evolve (as slowly as possible) to maintain viability, until time \bar{t}_1 where the control can remain constant during a nonempty time interval: *second punctuated equilibrium phase*.



Charles Darwin

However, they were anticipated by Darwin himself:

Addition to Chapter XI of the sixth edition of

Origin of Species by Charles Darwin

“Summary of the preceding and present chapters”

I have attempted to show that the geological record is extremely imperfect ; that only a small portion of the globe has been geologically explored with care ; that only certain classes of organic beings have been largely preserved in a fossil state ;

that the number both of specimens and of species, preserved in our museums, is absolutely as nothing compared with the number of generations which must have passed away even during a single formation ; that, owing to subsidence being almost necessary for the accumulation of deposits rich in a fossil species of many kinds, and thick enough to outlast future degradation, great intervals of time must have elapsed between most of our successive formations ; that there has probably been more extinction during the periods of subsidence, and more variation during the periods of elevation, and during the latter the record will have been least perfectly kept ; that each single formation has not been continuously deposited ; that the duration of each formation is, probably short compared with the average duration of specific forms ; that migration has played an important part in the first appearance of new forms in any one area and formation ;

that widely ranging species are those which have varied most frequently, and have oftenest given rise to new species ; that varieties have at first been local ; **and lastly, although each species must have passed through numerous transitional stages, it is probable that the periods, during which each underwent modification, though many and long as measured by years, have been short in comparison with the periods during which each remained in an unchanged condition.** These causes, taken conjointly, will to a large extent explain why though we do find many links- we do not find interminable varieties, connecting together all extinct and existing forms by the finest graduated steps. It should also be constantly borne in mind that any linking variety between two forms, which might be found, would be ranked, unless the whole chain could be perfectly restored, as a new and distinct species ; for it is not pretended that we have any sure criterion by which species and varieties can be discriminated”

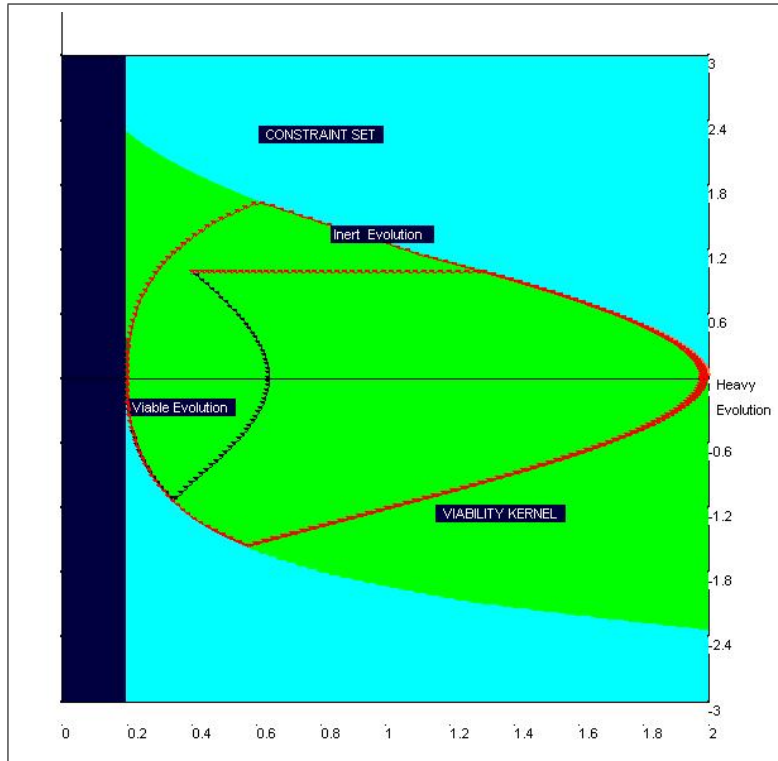


Figure 2: Viability Kernel and Inert Evolution. By clicking on Figure 2, the Saint-Pierre Viability Kernel Algorithm computes the viability kernel (which is the graph of the regulation map U_c) on a sequence of refined grids, provides an arbitrary viable evolution, the heavy evolution minimizing the velocity of the controls and which stops at equilibrium b , and the inert evolutions going back and forth from a to b in a hysteresis cycle. The graph of the inert evolution is shown in Figure ?? . Programme by Patrick Saint-Pierre

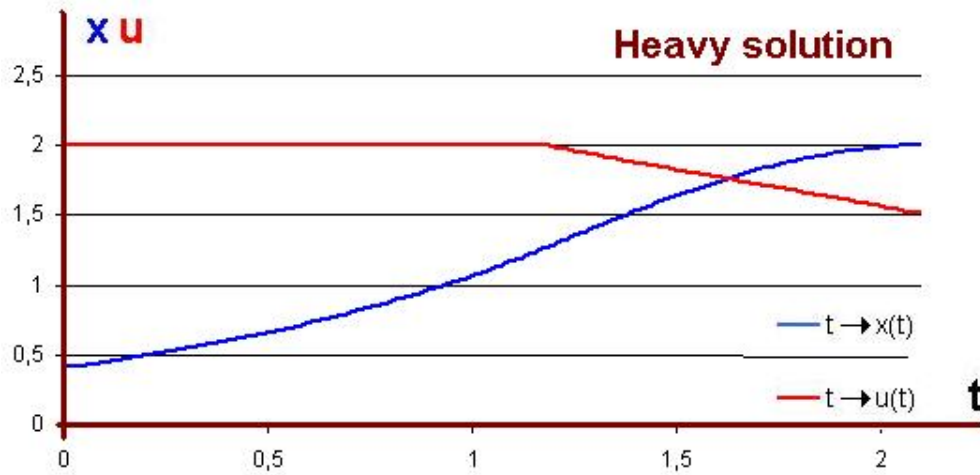


Figure 3: Graph of the Heavy Evolution. Both the graphs of the heavy evolution (in blue) and of its control (in red) are plotted. They are not computed from the analytical formulas given below, but extracted from the Viability Kernel Algorithm. The control remains constant until the trajectory of the exponential solution hits the boundary of the viability kernel and then slows down when it is controlled with a decreasing linear time dependent controls with velocity equal to $-c$. It reaches in finite time the boundary of the constrained interval with a velocity equal to 0 and may remain at this equilibrium. Source: Patrick Saint-Pierre.

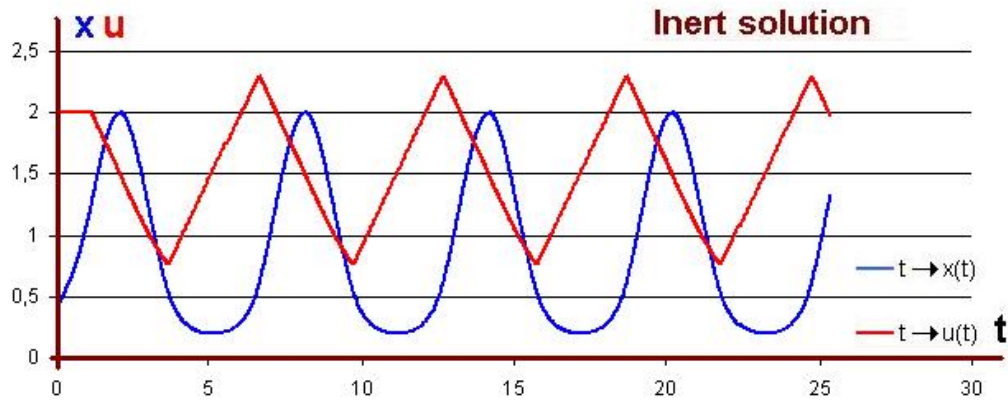


Figure 4: Graph of the Inert Evolution. Both the graphs of the inert evolution (in blue) and of its control (in red) are plotted. They are not computed from the analytical formulas given below, but extracted from the Viability Kernel Algorithm. The **velocity** of the control remains constant until the trajectory of the solution hits the boundary of the viability kernel and then switches to the other extremal control with opposite sign and so on. The evolution is then periodic, alternatively increasing and decreasing from the lower bound of the constrained interval to its upper bound. Source: Patrick Saint-Pierre.

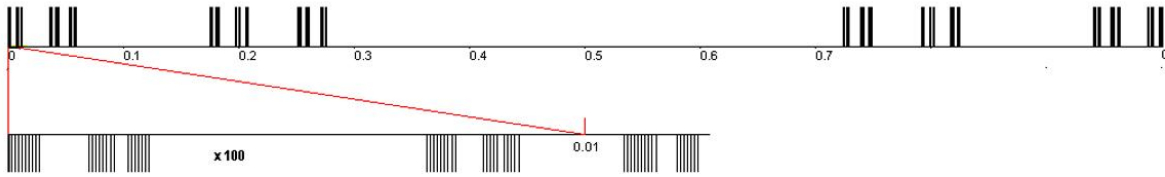


Figure 5: **Viability Kernel of $[0, 1]$ under the Quadratic Map $\Phi(x) := \{rx(1 - x)\}$** where $r > 4$, which is a Cantor set. The interval $[0, 1]$ has been thickened for easier visibility of the viability kernel. The discrete system $x_{n+1} = rx_n(1 - x_n)$ is the “*discrete analogue*” of the Verhulst logistic equation $x'(t) = rx(t)(1 - x(t))$ under which the interval $[0, 1]$ is equal to its viability kernel. This illustrates the crucial fact that *viability properties of the discrete analogues of continuous-time systems can be drastically different*, and that there is some danger when one has to choose between discrete and continuous time when we model dynamical phenomena. Source: Patrick Saint-Pierre.

1 Motivations & Comments

There should be no difficulty in accepting the idea of an environment on which the cognitive systems act.

Further studies should make precise the structure of the space describing the environment. It should be at least represented as the product of a physical environment, a biological one, formed of other species (which thus becomes a dynamical environment by involving the relations with other organisms) and a conceptual environment, formed of cultural artifacts available to human brains (Popper's third world). In the case of pluricellular organisms, one can consider that the internal organization is a part of the environment of the system, whose objective is the survival of organism (the receptor cells of the internal states are called the proprioceptor cells). For the sake of simplicity, we represent this too complex a structure by a finite dimensional vector-space.

One of the universal properties of living matter is to consume resources provided by the environment and thus, to transform, create or destroy it in order to keep alive their organism and to transfer their genes to their offsprings.

(Some four billion years ago the photosynthesis of the first organisms transformed the existing atmosphere of methane and ammonia to the oxygenated one we know today –this was probably the first example of pollution ! –) For that purpose, they must perceive in the environment sources of nutrients and to have access to them, in a way or another. From perception to action, most organisms process a very complex sequence of chemical communications, one molecule leaving an emitter cell and arriving to a receptor cell, modifying its chemical composition.

This chemical communication process, by sending macromolecule from an emitter cell to a receptor cell, the emission and the reception of such a molecule modifying the chemical properties of these cells, could constitute a relevant framework to many biological phenomena at several levels.

One can consider “**adaptive systems**” as systems formed of receptors (of the environment), effectors (on the environment) and intermediate (or processing) cells. Their behavior consists in acting at each instant to bring an unfavorable state of the environment to a more favorable one.

The perception of the environment by the emitters provoke the transmission of molecular messages towards the cells of the system.

These messages go from one cell to the others, the reception by an intermediate cell inducing the emission of another message.

The action starts after the reception of the message by the effectors, modifying the state of the environment which is again perceived by the system and leads to a new action by the system.

At this level, the endocrine system and the nervous system may be distinguished by the **mode of transport** of the chemical messenger. This transport is slow and non-specific in the case of the endocrine system : hormones are carried by the blood; it is fast and specific in the case of the nervous system : neurotransmitters have to cross a synapse (the place where the axon of a “**presynaptic**” neuron meets the dendrite of a “**postsynaptic**” one), which is only 0.02μ wide. When the pulse-coded information sent by the postsynaptic neuron reaches a certain threshold value, it releases neurotransmitters in the synapse, inducing an electrical response on the postsynaptic membrane after about 10^{-3} seconds.

There should also be no problem in accepting the existence of cerebral activity which operates the internal organs of the body and the muscular activity by which interaction with the environment is possible.

The existence of conceptual controls and their use in a recognition mechanism are more questionable assumptions.

First, nothing forbids in principle the synaptic weights to play the role devoted to the conceptual controls. But this would probably requires pretty large variability margins (if evolution of the learning is required). Second, this representation isolates nervous systems from the endocrine systems, for which the concept of “**weight**” between an emitter cell and a receptor cell is difficult to conceive. Maybe it would be cautious to leave open the problem of the precise nature of those conceptual controls, even if we accept to postulate their existence.

1. The ambiguous concept of **perception** includes both an “**objective**” and a “**subjective**” component. The objective component, which we call sensory perception, is provided by the neuronal circuit activated by the sensory receptors. But everyone knows that there is also a subjective component by which this sensory perception is interpreted : this interpretation may depend on many factors (previous experiences, emotional state, attention level, etc.), i.e., on a state of cerebral activity independent of the sensory inputs. This independent activity represents part of the regulatory control which we called conceptual control.

2. If we accept the existence of an endogenous cerebral activity which “**interprets**” the sensory perception of the environment, we must postulate the existence of a recognition mechanism which tells us whether a conceptual control and the sensory perception of the environment and its variations are consistent. It seems that brains have evolved during phylogenesis systems which transform information on bodily needs and environmental events into cerebral activity producing either pleasure (comfort) or pain (discomfort). These systems are known by psychologists as **motivational systems**, and are naturally more sophisticated than strictly pleasure-seeking or pain-avoiding systems. They include the emotional system and the homeostatic drive systems, which basically keep the organism functioning (for example, the hunger drive). They are often regulated by hormones.

These systems reveal the relation between the perception of the environment and the conceptual controls: if these are not consistent the situation can be remedied by:

- acting on the environment
- changing the conceptual control when action on the environment consistent with the existing conceptual control.

The latter strategy (change of conceptual controls) appears to be less frequent than the first and, for many subsystems (such as the homeostatic systems), is quite impossible. This is probably due to an **inertia principle** which we will postulate later, which states that whenever a conceptual control “works” (i.e., allows to keep the evolution of the cognitive system viable), we keep it.

3. The idea of a recognition mechanism based on conceptual controls is consistent with the concept of **epigenesis**. The recognition mechanism outlined above is basically a selection mechanism with a definite Darwinian flavor, choosing conceptual controls as a function of the environment and changes in the environment. By representing the cerebral activity as the flux of neurotransmitters in individual synapses (see below), one could suppose that the synapses used most frequently would be stabilized, while those used less frequently would deteriorate.

But the mere description of the synapses which are stabilized after a period of activity is capable of explaining epigenesis only to the extent that a road network can determine the routes taken by cars - in this case, one can agree to observe that if an existing network constrains the traffic without determining it, this is the existing travel patterns which governs the evolution of the network by requiring the maintenance of commonly used routes while the others can be neglected. This is the reason why we have chosen to study the evolution of “cars” at the crossroads of the network to infer its evolution, instead of to study this network per se.

4. We also postulated that the evolution of the recognition mechanism is programmed genetically. This recognition mechanism is probably rather simple in its principle: it may just open or close (activate or deactivate) a number of neuronal circuits during one or several specified periods of time, allowing both the neurotransmitters released by the perception of the environment and the conceptual controls to pass through circuits allowing their comparison.

It seems likely that some components of this mechanism are **periodic** with overlapping periodicity. These components are the many biological clocks involved in maintaining the homeostatic equilibrium of the organism

It may be postulated that the recognition of the periodicity of the trajectories of the sun and the moon by periodic components of the recognition mechanism in combination with suitable conceptual controls leads to the perception and concept of time.

These periodic components of the recognition mechanism probably lie at the heart of the ability of learning rules **to recognize regularities supplied by these cyclic processes by “analogous” endogenous cyclic processes** and then, to **“memorize them by auto-sustaining”** these cyclic processes, to extrapolate them, as well as to attempt to look for causal relations.

Comparison of trajectories with different periods may provide reading mechanisms which are either retrograde (recall processes) or anterograde (anticipation processes).

Memorized phenomena can be reconstructed from initial conditions, which can be pieces of time-dependent functions to neurotransmitters (and/or hormones) by (set-valued) dynamical processes. **The set-valued character of the consequences of initial data could be as important as the support of the initial function is small.** The richer is the input, the more reduced is the uncertainty or the availability of the outputs.

This cyclic character allows also to recover the same sequence (piece of trajectory) from several different pieces of a periodic initial datum, which can range over different geographical areas of the nervous system.

One can also take into account the mechanism of **generalization**: if two time dependent functions coincide on a given time interval, one could say that the common piece of these functions is an abstraction of each of them. The difficulty to travel time backwards two functions to discover their common piece may explain the **difficulties of abstraction (or induction), compared to deduction which would amount to let go forwards the dynamic process** (whose set-valued character opposes the deterministic rigor to which dream the deduction professionals!.)

One can also revisit in this framework the problems studied through “**semantic networks**” , by replacing the representation of concepts by the nodes of the semantic network by the representation of concepts as (multivalued) concatenation of elementary functions from the time line to the set of synapses.

5. Other components of this mechanism are not cyclic, but are active only during a certain period. This may be illustrated by the phenomenon of “**imprinting**” in ethology : in animal species where the young are able to walk almost immediately after birth, the new-born animals follow the first moving object that they perceive, whatever this may be. (In practice, it is usually a parent.)

However, this susceptibility does not last indefinitely and involves the perception of movement.

For example, ducklings can be imprinted only during the first twenty-four hours of their life, with sensitivity at a maximum between the 14th and 17th hours. The crucial factor in imprinting is the mobility of the object to be imprinted, and this reveals the importance of the perception of variations in the environment.

6. The assumption of a recognition mechanism explain the **adaptability** and **redundancy** of cerebral activities. A cognitive system can recognize the same sensory perception using different conceptual controls at different times – this is redundancy. Thanks to the cyclic nature of many components of cerebral activities, this sensory perception can be “**interpreted**” in several ways, provoking different actions (since we have assumed that the action taken depends upon the conceptual controls) – and this is adaptability.

7. The components of the recognition mechanism based on one or a small number of conceptual controls operate the automatic biological systems (the automatic nervous system, etc.), since in this case the subsystem inherits the genetic program of the component of the recognition mechanism. Stimuli-Response (SR) processes belong to this class.

8. The concept of a recognition mechanism reflects the dichotomy between “**conceptually-driven processes**” (top-down) and “**data-driven (bottom-up) processes**” introduced by specialists in cognitive psychology and pattern recognition.

In this case the data-driven process is the cerebral activity provoked by the sensory perception of the environment while the conceptually-driven process takes the form of conceptual controls (this is the origin of our terminology).

9. The recognition mechanism is also consistent with the concept of **metaphor**, regarded as a combination of a sensory perception of the environment and conceptual control **recognized** by the recognition mechanism. A feeling of understanding, which amounts to a feeling of pleasure, occurs when a metaphor is recognized by the recognition process.

The set-valued character of the recognition mechanism is related to the mechanism of discovery allowed by the possibility to explore new conceptual controls to interpret the sensory perception.

Perhaps thought processes can also fit into this representation, since they involve setting up conceptual controls in the form of assumptions and then comparing them with the perception of the environment (including the cultural environment). This dynamical process of **making and matching** seems to be quite universal.

10. The proposed mathematical metaphor describes a **learning process** as a feedback relation which associates a set of (learnable) conceptual controls with each sensory-motor state. The larger the set of conceptual controls associated with a sensory-motor state, the less deterministic the learning process.

This role of sensory-motor in learning has been observed and emphasized by Piaget and others when they described the learning processes of children. Here, we characterize the viable learning processes and we deduce the existence of a largest learning process.

This role is consistent with several observed facts. For instance, studies of the imprinting phenomenon have shown that the greater the effort made by the young animal to follow the moving object, the stronger is the imprint. When one of the components of the sensory-motor state is suppressed, the learning mechanism does not work normally.

For instance, if kittens are raised in a visual environment composed of black and white vertical lines, they are unable to “see” horizontal stripes later in life.

In another experiment, two kittens from the same litter spend several hours a day in a contraption which allows one kitten fairly complete freedom to explore and perceive its environment while the other is suspended passively in a “gondola” whose motion is controlled by the first kitten. Both animals receive the same visual stimulation, but the active kitten learns to interpret these signals to give it an accurate picture of its environment while the passive kitten learns nothing and is, in practical terms, “blind” to the real world.

2 Recognition Mechanisms

It is possible to conceive in an abstract way recognition mechanisms between two circuits of neurotransmitters.

Let us consider for example two subnetworks of networks with the same number of synapses through which circulate two circuits of neurotransmitters. For comparing these two circuits, we introduce a third subnetwork with the same number of synapses. It is enough that from each synapse of the same rank of each of the two subnetworks branch neurons — one excitatory the other one inhibitory — connected to the synapse of the same rank of the third subnetwork.

Figure 6: Abstract recognition Mechanism

Consider now two circuits of neurotransmitters circulating in each subnetwork. They influence through the branching neurons the synapse of the third subnetwork with antagonist effects. It is sufficient that the synapses of the third circuit are silent — excitation produced by one circuit is inhibited by the other circuit — for these two circuits of neurotransmitters to be recognized.

Furthermore, when these two circuits are not recognized, the synapses of the third circuit become excited. The farther apart are the two circuits, the more active is the third circuit. One can imagine that this third circuit, when it is active, triggers the activity of other circuits which eventually, induce some action on the environment in order that one of the circuit becomes recognized by the other.

The same type of mechanism can be used to recognize time dependent functions of neurotransmitters journeying through a given synapse by another time dependent function of neurotransmitters going through a second synapse. Assume that each is a discrete function of n periods of time. We introduce a subnetwork of n neurons and we link each of the two neurons to the synapses of the subnetwork with antagonist effect (if one neuron arriving at a given synapse is excitatory, the other one is inhibitory and vice-versa). Assume furthermore that the duration of transit of the influx from each of the two neurons to the j th neuron of the subnetwork is equal to j units of time.

In this case again, if each synapse of the subnetwork is silent, one can say that the two functions passing through each of the neurons are recognized. The activity in the subnetwork is a measure of the lack of recognition of these two functions. This activity can then trigger other subnetworks and finally, the action of the system on the environment.

Namely, we denote by $(a_{-n}, a_{-n+1}, \dots, a_{-1})$ a circuit of neurotransmitters perceived by only one neurone a and by $(b_{-n}, b_{-n+1}, \dots, b_{-1})$ an endogenous circuit passing through a neurone b to be compared at time 0. We assume for simplicity that the time is discrete between the first instant $-n$ and the n th, which is -1 . The “**comparing circuit**” is made of n neurones, denoted by $(r_{-n}, r_{-n+1}, \dots, r_{-1})$, of n acting neurones (or chains of neurones) p_j linking “**receptive neurone**” a to neurones r_j and of n inhibitor neurones n_j linking the endogenous comparing neurone b to these neurones r_j ($j = 1, \dots, n$). The duration of the “**propagation**” from the receptive neurone a and from comparing neurone b to neurone r_j along neurones p_j and n_j equals j time units.

If at time 0 the “sum” of excitations and of inhibitions received by the n neurones r_j (triggered by exogenous circuits $(a_{-n}, a_{-n+1}, \dots, a_{-1})$ and endogenous circuits $(b_{-n}, b_{-n+1}, \dots, b_{-1})$ after transiting neurons p_j and n_j respectively is equal to zero, then we have a process recognizing these two circuits. If not, each of the neurones r_j will have a (positive or negative) activity.

If we assume furthermore that starting from the circuits r_j “**effector neurones**” c_k^j controlling the actions of the cognitive system on the environment, we obtain a simple system describing how an organisms remains homeostatic as long as the perception of the environment is recognized by an endogenous (when neurones c_k^j are not excited, no action is triggered), whereas a disequilibrium between the perceived circuit and the endogenous circuit triggers actions depending on the discrepancy measured at each neuron r_j .

3 Viable Learning Processes

3.1 Sensory-Motor States and Conceptual Controls

We represent the **environment** by a finite-dimensional vector space X .

We then assume that **the state of cerebral activity is described by the evolution of neurotransmitters in each synapse** .

We denote by S the set of synapses and by $C := \mathbf{R}^S$ the **cerebral space**. The component y_s of an element $y = (y_s)_{s \in S}$ of this cerebral space C denotes the number of neurotransmitter molecules passing through synapse s (with a minus sign if the role of the neurotransmitter is inhibitor). We shall describe the temporal cerebral activity by several functions of time into the cerebral space $C := \mathbb{R}^S$.

We distinguish two classes of time-dependent functions:

1— The function $t \mapsto a(t) \in C$ which describes the evolution of **cerebral motor activity**, i.e., the knowledge, in each synapse $s \in S$, at each time t , the of the number $a_s(t)$ of neurotransmitters involved in the perception process.

2— The function $t \mapsto c(t) \in C$ which describes the endogenous evolution of the **conceptual controls**, i.e., the knowledge, in each synapse $s \in S$, at each time t , of the number $c_s(t)$ of neurotransmitters involved in the conceptual activity (which has to be compared with the other cerebral functions in the recognition mechanism).

Hence these two functions $a(\cdot)$ and $c(\cdot)$ determine at each time t the active **“neuronal networks”** and their evolution.

With each function $t \mapsto z(t) \in C$, we can associate its “neuronal trace” $N(z(t))$ defined by

$$N(z(t)) := \{ s \in S \mid z_s(t) \neq 0 \}$$

which specifies the set of active synapses.

We did not choose to emphasize the topological nature of the neural network, since it can be described in terms of the “traces” left by the circulation of neurotransmitters; a given synapse is “weighted” by the total number of neurotransmitters crossing it during each period.

There is also few hopes to obtain through “functional” logical reasoning an explanation of the topological structure of nervous system in large units without a thorough study of the apparitions of these structures in phylogenesis.

The **states** of the systems are therefore sensory-motor states (x, v, a) ranging over the vector space $X \times X \times C$.

The **controls** of the system are elements c of C .

3.2 The Recognition Mechanism

The recognition mechanism compares **the sensory perception of the environment, the sensory perception of its variation, and the state of the conceptual control at each instant.**

We shall describe it by a family of subsets $\mathcal{R}(t)$ of the space $C \times X \times X$. There is recognition at instant t of the state of the environment x and its variation v by the conceptual control c if we have

$$(c, x, v) \in \mathcal{R}(t) \tag{1}$$

$\mathcal{R}(t)$ describe the set of possible **metaphors** (c, x, v) between a conceptual control and the sensory perception of the environment which can be recognized at time t .

We can regard $\mathcal{R}(t)$ as the graph of the set-valued map $R(t) : X \times X \rightsquigarrow C$ associating with the state of the environment x and its variation v at time t the subset $R(t; x, v)$ of conceptual controls c which can **recognize** the pair (x, v) .

This is the **“data driven”** version of the recognition mechanism. The inverse $R^{-1}(t; \cdot)$ associates with any conceptual control c the set of pairs (x, v) which can be recognized by c . This is the **“conceptually-driven”** version of the recognition mechanism.

3.3 The Viability Constraints

The viability constraints translate the fact that the cognitive system consumes scarce resources of the environment in order to maintain and improve through a recognition mechanism an internal state (which tends to be deteriorate) with respect to a reference state (which may evolve with time).

We translate into a quite general form these constraints to which the cognitive system must adapt by requiring that

$$\forall t \geq 0, \quad a(t) \in K(t, x(t)) \quad (2)$$

where K is a set-valued map from $]0, \infty] \times X$ to C .

In other words, this set-valued map K specifies what are the states of the environment which allow the cognitive system to survive and the constraints which are set on his motor activity.

3.4 The Action Law

We assume that the evolution of the environment due to the action of the cognitive system depends upon both the environment and the cerebral motor activity of the cognitive system. At this level of generality, we can assume that it is governed by a second order differential equation of the form

$$x''(t) = f(t, x(t), x'(t), a(t)) \quad (3)$$

where f is a map from $[0, \infty[\times X \times X \times C$ to X .

3.5 The Perception Law

The evolution of the cerebral motor activity induced by the perception of the environment and regulated by conceptual controls is governed by a controlled differential equation: the velocity of the flux of neurotransmitters involved in the motor activity depends at each time t not only upon the perception of the environment and the cerebral motor activity, **but also upon the conceptual control which is currently active.** Hence, we can write that

$$a'(t) = g(t, x(t), x'(t), a(t), c(t)) \quad (4)$$

where g is a from $[0, \infty[\times X \times X \times C \times C$ to C .

The cerebral activity induced by sensory perception of the environment (both external and internal) is included implicitly in the perception law, which also takes into account the propagation of synaptic excitation along the set S of synapses

3.6 The Learning Process

Once the viability constraints and the recognition mechanism are given, we can formulate their compatibility by introducing the following set-valued map P defined by

$$P(t, x, v, a) := \begin{cases} R(t, x, v) & \text{if } a \in K(t, x) \\ \emptyset & \text{if } a \notin K(t, x) \end{cases} \quad (5)$$

and require that the domain of this set-valued map P is not empty.

Definition 1 (Learning Processes) *A **learning process** is a set-valued map L with closed graph which at each time t associates with every sensory-motor state (x, v, a) a subset $L(t, x, v, a)$ of conceptual controls, which is consistent with the recognition mechanism (1) and the viability constraints (2) in the sense that:*

$$\forall (t, x, v, a) \in \text{Dom}(P), \quad L(t, x, v, a) \subset P(t, x, v, a) \quad (6)$$

The “**growth rate**” $\rho \geq 0$ of a learning process is regarded as a parameter.

Definition 2 *We shall say that a closed ρ -learning process L_ρ enjoys the **viability property** (with respect to the evolution laws (3) and (4)) if and only if for any initial time t_0 and any initial state $(t_0, x_0, v_0, a_0, c_0)$ in the graph of the learning process, there is a solution*

$$t \in [t_0, \infty[\mapsto (x(t), a(t), c(t))$$

to the system of differential inclusions

$$\begin{cases} i) & x'' & = & f(t, x(t), x'(t), a(t)) \\ ii) & a'(t) & = & g(t, x(t), x'(t), a(t), c(t)) \\ iii) & \|c'(t)\| & \leq & \rho(\|c(t)\| + 1) \end{cases} \quad (7)$$

satisfying

$$\forall t \geq t_0, \quad c(t) \in L_\rho(t, x(t), x'(t), a(t)) \quad (8)$$

When $\rho = 0$, differential inclusion (7)iii) implies that the conceptual control must remain constant. Equation (8) specifies that for any $(t_0, x_0, v_0, a_0, c_0)$ of the graph of L_0 , one must have

$$\forall t \geq t_0, c_0 \in L_\rho(t, x(t), x'(t), a(t))$$

which can be rewritten in the form

$$\forall t \geq t_0, (x(t), x'(t), a(t)) \in L_0^{-1}(t; c_0)$$

Hence $c \rightsquigarrow L_0^{-1}(t; c)$ yields at each time t_0 the subset of sensory-motor states (x_0, v_0, a_0) which are “**learnable**” by the conceptual control c_0 at time t :

there exists a solution to the system of differential equations

$$\begin{cases} i) & x''(t) = f(t, x(t), x'(t), a(t)) \\ ii) & a'(t) = g(t, x(t), x'(t), a(t), c_0) \end{cases}$$

starting at (x_0, v_0, a_0) and satisfying

$$\forall t \geq 0, (x(t), x'(t), a(t)) \in L_0^{-1}(t; c_0)$$

$t \rightarrow L_0^{-1}(t; c)$ is the **“sensory-motor tube”** .

$$\begin{aligned} & \text{Set } M_\rho(t, x, v, a, c) \\ & := DL_\rho(t, x, v, a, c)(1, v, f(t, x, v, a), g(t, x, v, a, c)) \end{aligned}$$

Theorem 3 *Assume that f and g governing the Action Law and the Perception Law of the cognitive system are continuous and have linear growth, that the graphs of $(t, x, v) \rightsquigarrow R(t, x, v)$ and $(t, x) \rightsquigarrow K(t, x)$ are closed. Then a ρ -learning process L_ρ enjoys the viability property if and only if $\forall (t, x, v, a, c) \in \mathcal{L}_\rho$,*

$$M_\rho(t, x, v, a, c) \cap \rho(\|c\| + 1)B \neq \emptyset \quad (9)$$

*Furthermore, there exists a **largest viable** ρ -learning process, and thus, largest sensory-motor cells.*

Proof — We apply Viability Theorem, for example) to the following system of differential inclusions defined on the closed subset (which is the graph of the set-valued map P defined by (5))

$$\mathcal{K} := \{ (t, x, v, a, c) \mid a \in K(t, x) \ \& \ c \in R(t, x, v) \}$$

by

$$\left\{ \begin{array}{l} i) \quad \tau'(t) = 1 \\ ii) \quad x'(t) = v(t) \\ iii) \quad v'(t) = f(t, x(t), v(t), a(t)) \\ iv) \quad a'(t) = g(t, x(t), v(t), a(t), c(t)) \\ v) \quad c'(t) \in \rho(\|c(t)\| + 1)B \end{array} \right.$$

We observe at once that a set-valued map L_ρ is a ρ -learning process if and only if its graph \mathcal{L}_ρ is contained in \mathcal{K} and that the ρ -learning process L_ρ enjoys the viability property if and only if its graph \mathcal{L}_ρ enjoys the viability property for the above system of differential inclusions.

The Viability Theorem states that \mathcal{L}_ρ enjoys the viability property if and only if it is a viability domain in the sense that

$$\begin{cases} \forall (\tau, x, v, a, c) \in \mathcal{L}_\rho, \exists w \in \rho(\|c\| + 1)B \\ (1, v, f(t, x, v, a), g(t, x, v, a, c), w) \in T_{\mathcal{L}_\rho}(t, x, v, a, c) \end{cases}$$

Taking into account the definition of the contingent derivative of L_ρ , this is the definition of a viable ρ -learning process.

Consequently, the learning process L_ρ enjoys the viability property if and only if it is viable.

We also know that there exists a largest closed viability domain (possibly empty) $\widetilde{\mathcal{L}}_\rho$ contained in \mathcal{K} , which is then the graph of the largest learning process \widetilde{L}_ρ . \square

The fundamental question arises: How can we select the evolution in a nondeterministic viable ρ -learning process L_ρ ?

We shall present a selection mechanism obeying the “**inertia principle**”, which states that whenever a conceptual control “**works**” (i.e., allows to keep the evolution of the cognitive system viable), we keep it.

We can translate this principle by saying that we choose to keep the velocity $c'(t)$ of the conceptual control equal to zero whenever it is possible.

One way (among others that we do not describe here) to implement this principle is to select heavy viable solutions, which minimize at each instant the norm $\|c'(t)\|$ of the velocity of the conceptual controls.

The conceptual controls evolve according the differential inclusion

$$c'(t) \in M_\rho(t, x(t), x'(t), a(t), c(t)) \quad (10)$$

The minimal selection $m_\rho(t, x, v, a, c) \in M_\rho(t, x, v, a, c)$

$$\|m_\rho(t, x, v, a, c)\| = \inf_{m \in M_\rho(t, x, v, a, c)} \|m\|$$

Theorem 4 *We posit the assumptions of Theorem 3 and we assume that the set-valued map M_ρ is lower semicontinuous with closed convex images. Then for every initial time t_0 and initial state (x_0, v_0, a_0, c_0) , there exists a solution to the system of differential equations*

$$\begin{cases} i) & x''(t) = f(t, x(t), x'(t), a(t)) \\ ii) & a'(t) = g(t, x(t), x'(t), a(t), c(t)) \\ iii) & c'(t) = m_\rho(t, x(t), x'(t), a(t), c(t)) \end{cases} \quad (11)$$

which satisfies

$$\forall t \geq 0, \quad c(t) \in L_\rho(t, x(t), x'(t), a(t)) \quad (12)$$

Definition 5 *The solutions to the system (11) of differential equations are called ρ -heavy solutions to the cognitive system.*

Observe that if at time t_1 , a heavy solution satisfies

$$(x(t_1), x'(t_1), a(t_1)) \in L_0^{-1}(t_1; c(t_1))$$

then we know that the conceptual control $c(t_1)$ can remain constant after t_1 , and that the state of the cognitive system will evolve for $t \geq t_1$ according the law

$$\begin{cases} i) & x''(t) = f(t, x(t), x'(t), a(t)) \\ ii) & a'(t) = g(t, x(t), x'(t), a(t), c(t_1)) \end{cases}$$

while remaining in the sensory-motor cell

$$\forall t \geq t_1, (x(t), x'(t), a(t)) \in L_0^{-1}(t; c(t_1))$$

If this happens, one can say that the conceptual control c_1 is a “punctuated equilibrium” (in the sense of Elredge and Gould) since it remains constant whereas the state may evolve.

Cervello

Merci pour votre
Thanks for your
Attention

